achildsguideto

www.achildsguideto.com

Final Push Maths 1

Foundation Paper Revision

Answers

Mr H

WWW.ACHILDSGUIDETO.COM

NUMBER OPERATIONS: LONG MULTIPLICATION AND LONG DIVISION

1 421 × 8.6

3620.6

421
8.6×
2526
33680
3620.6

2 12.56 × 7.3

91.688

12.56
7.3×
3768
87920
91.688

3 80654 ÷ 2.4

$33605\frac{20}{24} = 33605\frac{5}{6}$

33605
24 | 806540
−72
86
−72
145
−144
14
−0
140
120
20

1	24
2	48
3	72
4	96
5	120
6	144
7	168
8	192
9	216
10	240

4 62.234 ÷ 36

1.72872

1.728722
36 | 62.234000
−36
262
252
103
72
314
−288
260
−252
80
72
80

1	36
2	72
3	108
4	144
5	180
6	216
7	252
8	288
9	324
10	360

VECTORS

1 $\boldsymbol{a} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$

Find the solution to 3**a** + **b**

$$3\begin{pmatrix} 4 \\ 9 \end{pmatrix} + \begin{pmatrix} 7 \\ -1 \end{pmatrix} = \begin{pmatrix} 12 \\ 27 \end{pmatrix} + \begin{pmatrix} 7 \\ -1 \end{pmatrix} = \begin{pmatrix} 19 \\ 26 \end{pmatrix}$$

2 $\boldsymbol{a} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$

Find the solution to 7**a** - 2**b**

$$7\begin{pmatrix} -3 \\ 5 \end{pmatrix} - 2\begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} -21 \\ 35 \end{pmatrix} - \begin{pmatrix} 14 \\ 8 \end{pmatrix} = \begin{pmatrix} -35 \\ 27 \end{pmatrix}$$

3 $\boldsymbol{a} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$

Find the solution to 6**a** + 3**b**

$$6\begin{pmatrix} 7 \\ 6 \end{pmatrix} + 3\begin{pmatrix} 0 \\ -5 \end{pmatrix} = \begin{pmatrix} 42 \\ 36 \end{pmatrix} + \begin{pmatrix} 0 \\ -15 \end{pmatrix} = \begin{pmatrix} 42 \\ 21 \end{pmatrix}$$

BEST BUYS

1 Fred sells 6 chocolate bars for £2.40. Joanne sells 5 chocolate bars for £1.99.
Amir needs to buy 60 chocolate bars. Who provides the best value for money?

$6\overline{)2.40} = 0.40$ $\quad 5\overline{)1.99} = 0.39\frac{4}{5}$

Joanne provides the best value

2 Three taxi companies operate in a town. Their prices are as follows:

Cars n Cabs	ABC Carriages	Green Taxis
Collection £1.80	Collection £2.50	Collection £4.00
55p per mile	43p per mile	28p per mile

David wants to travel 12 miles.
Which company offers the cheapest price?

$(12 \times 0.55) + 1.80 = £8.40$

$(12 \times 0.43) + 2.50 = £7.66$

$(12 \times 0.28) + 4.00 = £7.36$

Green Taxis

3 A coat costs £180 in a shop. Online, the same coat can be purchased for $350 plus $24 postage and packaging.
The exchange rate is £1.00 : $2.07.
Where is the best place to buy the coat?

~~£180~~ $£180 \times 2.07 = \$372$

$\$350 + 24 = \$374$

Purchase in shop is cheaper.

EXPANDING AND FACTORISING

1 Expand $8(9x + 2)$ $= 72x + 16$

	$9x$	$+2$
8	$72x$	$+16$

2 Expand $3x(4x - 7)$ $= 12x^2 - 21x$

	$4x$	-7
$3x$	$12x^2$	$-21x$

3 Expand and simplify $(4x + 3)(5x - 2)$ $= 20x^2 + 7x - 6$

	$4x$	$+3$
$5x$	$20x^2$	$+15x$
-2	$-8x$	-6

4 Factorise $6x^3 + 15x^2 - 12x$

$3x(2x^2 + 5x - 4)$

5 Factorise $28x^2 - 21$

$7(4x^2 - 3) = 7(2x + \sqrt{3})(2x - \sqrt{3})$

6 Expand and simplify $6(3x + 2) - 2(5x - 3)$

$18x + 12 - 10x + 6 = 8x + 18$

SIMLUTANEOUS EQUATIONS

1 Solve for x and y.

$6x - 3y = 12$ —(i)
$8x + y = 46$ —(ii)

Multiply (ii) x 3

$24x + 3y = 138$ —(iii)

Add (i) + (iii)

$30x = 150$

$x = \frac{150}{30}$

$\therefore x = 5$

Substitute in (i)

$6(5) - 3y = 12$

$30 - 12 = 3y$

$\therefore y = \frac{30-12}{3}$

$= 6$

Check in (ii)

$8x + y = 46$

$8(5) + 6 = 46$ ✓

2 Solve for x and y.

$3w + 7v = 48$ —(i)
$6w - 3v = 45$ —(ii)

Multiply (i) x 2

$6w + 14v = 96$ —(iii)

Subtract (iii) - (ii)

$17v = 51$

$\therefore v = \frac{51}{17}$

$v = 3$

Substitute in (i)

$3w + 7(3) = 48$

$\therefore 3w = 48 - 21$

$= 27$

$\therefore w = \frac{27}{3}$

$= 9$

Check in (ii)

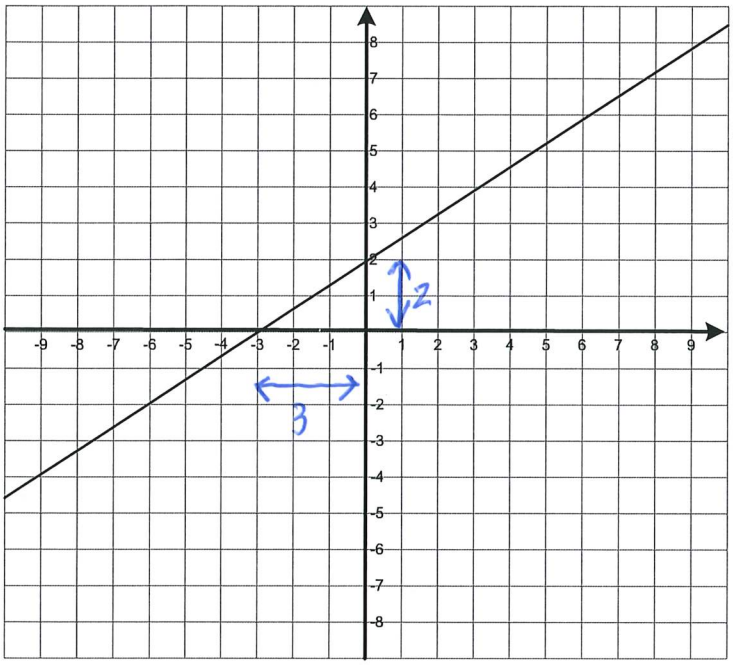
$6w - 3v = 45$

$6(9) - 3(3) = 45$

$54 - 9 = 45$

GRAPHS

1 L is a line segment shown on the graph below.
Write down the equation for line L.



$c = 2$

$m = \frac{\text{Rise}}{\text{Run}} = \frac{2}{3}$

$y = mx + c$

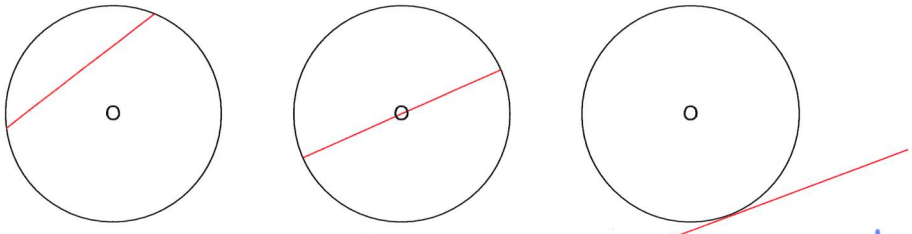
$\therefore \quad y = \frac{2}{3}x + 2$

2 Fill the missing values into the table below for the graph $y = x^2 - 5$ in the range $-3 \leq x \leq 3$.

x	-3	-2	-1	0	1	2	3
y	4	-1	-4	-5	-4	-1	4

CIRCLES

1 Name the lines shown on these circles in red.



Chord Diameter Tangent

2 The radius of a circle is 12cm. Calculate its circumference. Give an exact answer.

$C = 2\pi r$

$= 2 \times \pi \times 12$

$= 24\pi$ cm

3 The diameter of a circle is 16cm. Calculate its area giving the answer correct to 3 significant figures.

$r = \frac{d}{2} = \frac{16}{2} = 8$

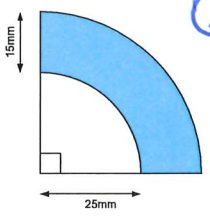
Area $= \pi r^2$

$= \pi \times 8 \times 8$

$= 64\pi$

$\approx 201.0619298 \approx 201$ cm^2

4 What is the area of the shaded section in the diagram below?



(1) Area $= \pi r^2$

$= \pi \times (15+25)^2$

$= 1600\pi$ mm^2

Area of Sector $= \frac{1600\pi}{4}$

$= 400\pi$ mm^2

(2) Area $= \pi r^2$

$= \pi \times 25 \times 25$

$= 625\pi$ mm^2

Area of Sector $= \frac{625\pi}{4}$

$= 156\frac{1}{4}\pi$ mm^2

(3) Area of shaded part

$= (400 - 156\frac{1}{4})\pi = 243\frac{3}{4}\pi$ mm^2

SEQUENCES

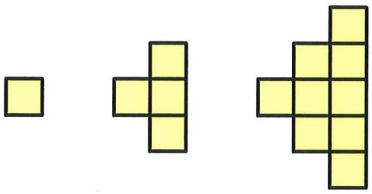
1 Below is the start of a sequence. Write the next three terms in the sequence.

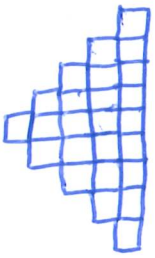
15, 11, 7, 3, ... , ... , ...

-1 -5 -9

2 Below is a sequence.

a Draw a diagram for the **fifth** term in the sequence.





b Is this sequence linear? Explain how you know.

1, 4, 9
+3 +5

No. The sequence is not linear.
It is likely to be quadratic.
It is not linear because the difference increases by 2 each time.

3 $\frac{4n+12}{5}$ is the nth term for a sequence.

What is the twenty-fifth term in the sequence?

$$\frac{4(25)+12}{5} = \frac{112}{5} = 22\frac{2}{5}$$

4 Below is a linear sequence.

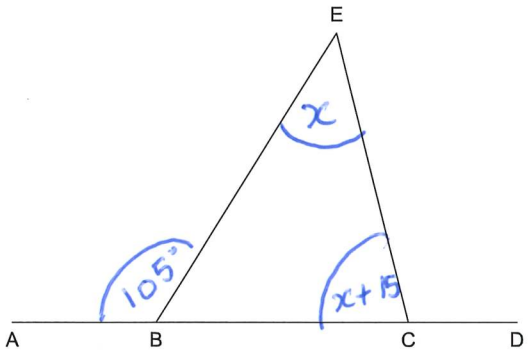
-12 -5 2 9 16
+7 +7 +7 +7

Find the nth term.

nth term $= 7n - 19$

GEOMETRY

1 AD is a straight line.
BCE is a triangle.



Angle ABE is 105°.
Angle BCE is 15° larger than angle BEC.
Find angle ECD.
Give reasons at each stage.

$\angle EBC = 180 - 105 = 75°$ (Angles on a straight line total 180°)

$x + x + 15 + 75 = 180$ (Angles in a triangle total 180°)

$\therefore 2x + 90 = 180$

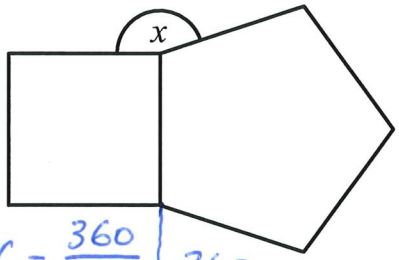
$\therefore 2x = 90$

$\therefore x = 45°$

$\angle ECB = 45 + 15 = 60°$

$\therefore \angle ECD = 120°$ (Ls on a straight line total 180°).

2 The diagram below shows a square and a regular pentagon.



Find the value of x.

Exterior Angle $= \frac{360}{4} = 90$

$\frac{360}{5} = 72$

$x = 90 + 72$

$= 162°$

Exterior Angle $= \frac{360}{n}$

where n is no. of sides.

RATIO

1 Bill, Mary and Jacob share 48 sweets in the ratio 7:5:4.
How many sweets does each person get?

7+5+4 = 16

48 ÷ 16 = 3 (one part)

Bill = 7 x 3 = 21 sweets

Mary = 5 x 3 = 15 sweets

Jacob = 4 x 3 = 12 sweets

2 John, Julie and Joel put some money into a pot. The ratio that they put in is 7:8:11.
Joel puts in £132. How much money is there in total?

132 ÷ 11 = £12

7 x 12 = 84

8 x 12 = 96

180

180 + 132 = £312

3 Beatrice, Ben and Billy play a game. The ratio of scores is 8:13:7. Billy scores 54 fewer points than Ben. How many points does Beatrice score?

13 - 7 = 6

54 ÷ 6 = 9 (one part)

8 x 9 = 72 points

Beatrice gets 72 points

4 Write the ratios 12:15:27 as fractions in their simplest form.

4:5:9

4+5+9 = 18

$\frac{4}{18} : \frac{5}{18} : \frac{9}{18} = \frac{2}{9} : \frac{5}{9} : \frac{1}{2}$

5 Write 7:12 in the form 1:n

÷7 (7 : 12) ÷7
(1 : 12/7)

1 : 1.714285...

6 The three little pigs built their houses of straw, sticks and bricks. The cost of single item of material was in the ratio 3:7:15. The amount of each material needed was in the ratio 3:2:7. The pig building his house of straw paid £270 for the materials for his house. How much did the other two pigs pay for the materials for their houses?

3 x 3 = 9

7 x 2 = 14

15 x 7 = 105

£270 ÷ 9 = £30

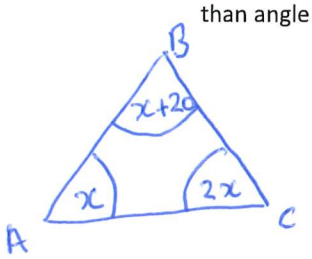
14 x 30 = £420 (sticks)

105 x 30 = £3,150 (bricks)

WRITING EQUATIONS FROM PROBLEMS

Geometry: With these, it might be helpful to draw a diagram. **It is essential to write the equation.**

1 A scalene triangle, ABC, has angle CAB which is half the size of angle BCA. Angle ABC is 20° more than angle CAB. Find the size of each angle.



$x + x + 20 + 2x = 180$ (Ls in a Δ total 180°)

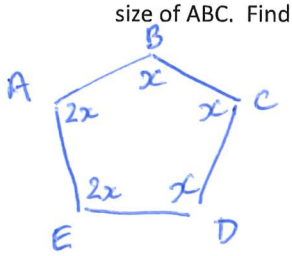
$4x + 20 = 180$

$4x = 160$

$x = 40$

So $\angle BAC = 40°$, $\angle ACB = 80°$, $\angle ABC = 60°$

2 A pentagon has angles ABC, BCD and CDE which are all equal. Angles DEA and EAB are twice the size of ABC. Find the size of each angle.



$7x = 540$

$\therefore x = \frac{540}{7} = 77\frac{1}{7}°$

$ABC = BCD = CDE = 77\frac{1}{7}°$

$DEA = EAB = 154\frac{2}{7}°$

Everyday problems

3 David thinks of a number. He adds seven to his number and then multiplies his result by 3. He now divides his answer by 6. He finishes with a number which is exactly twice the number of which he thought in the first place. What was David's original number?

$\frac{3(x+7)}{6} = 2x$

$\frac{3x+21}{2} = 2x$

$x + 7 = 4x$

$7 = 3x$

$\therefore x = \frac{7}{3}$

4 Charlie thinks of a number. He halves it and then adds the result to his first number. He halves the result again and adds that to his first number again. His answer is an integer. What is the lowest number of which Charlie could have originally thought?

$\frac{1}{2}\left(\frac{x}{2} + x\right) + x$

$\frac{x}{4} + \frac{x}{2} + x \in \mathbb{Z}$

So $x = 4$.

TWO WAY TABLES AND PROBABILITY

1 In a school, years 10 and 11 total 680 pupils, 120 pupils in Year 10 study history for their option one choice.
81 students in Year 11 study geography whilst 119 students in Year 10 study it.
Altogether, 295 students study history.
There are 334 students in Year 10.
There are 90 business studies students in Year 11.

a Complete the table below.

	Business Studies	Geography	History	Total
Year 10	95	119	120	334
Year 11	90	81	175	346
Total	185	200	295	680

b A student is chosen at random. What is the probability that the student is a Year 10 student studying Geography?

$$\frac{119}{680}$$

c Of the business studies students, a student is chosen at random. What is the probability that the student is in Year 11?

$$\frac{90}{185} = \frac{18}{37}$$

d Giving your answer to three decimal places, what percentage of students study business studies?

$$\frac{185}{680} \times 100 = 27.20588235$$

$$\approx 27.206$$

INDICES

1 What is the value of $7^0 \times 5$?

$1 \times 5 = 5$

2 What is $m \times m \times 2m \times 3m$?

$6m^4$

3 What is the value of $\left(\frac{3}{4}\right)^2$?

$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

4 What is $\frac{m^5 \times m^7}{m \times m^2 \times m^3}$?

$\frac{m^{12}}{m^6} = m^6$

5 Write $\frac{3 \times 3^3}{3^2 \times 3^7}$ in the form 3^n where n is an integer.

$\frac{3^4}{3^9} = 3^{-5}$

6 Write $\frac{128^3 \times 8^7 \times 64^5}{256^3 \times 32^5}$ in the form 2^n where n is an integer.

$\frac{(2^7)^3 \times (2^3)^7 \times (2^6)^5}{(2^8)^3 \times (2^5)^5} = \frac{2^{21} \times 2^{21} \times 2^{30}}{2^{24} \times 2^{25}} = \frac{2^{72}}{2^{49}} = 2^{23}$

PIE CHARTS

1 In a school, St James' School of the Arts, students are given the options of studying French, German, Italian or Spanish.

The table below shows how many students study each language.

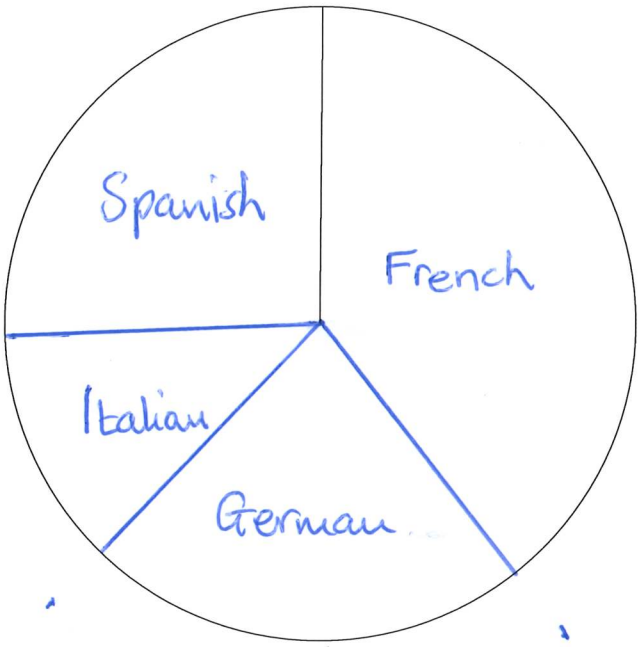
French	280
German	164
Italian	86
Spanish	190

$280 + 164 + 86 + 190 = 720$

$\frac{360}{720} = \frac{1}{2}$

$\frac{1}{2} \times 280 = 140$
$\frac{1}{2} \times 164 = 82$
$\frac{1}{2} \times 86 = 43$
$\frac{1}{2} \times 190 = 95$

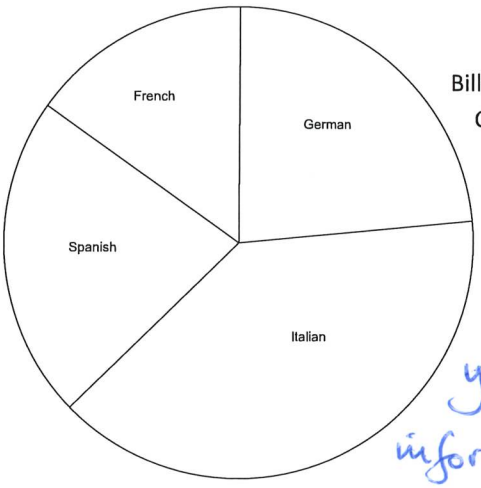
a Use the circle below to draw a pie chart displaying this information.



b What percentage of students study Spanish?

$\frac{190}{720} \times 100 = 26.38\%$

c Another school, St Cuthbert's High School, offers the same four subjects.



Billy says that more people study Italian at St Cuthbert's High School than at St James' School of the Arts.

d Is Billy correct? Explain how you know.

You cannot tell because you aren't told any numerical information and pie charts only show relative information.

FRACTIONS

1 Change $2\frac{7}{12}$ into an improper fraction. $\frac{(2\times12)+7}{12} = \frac{31}{12}$

2 Solve and simplify $\frac{9}{14} \times \frac{7}{15} =$ $\frac{\cancel{9}^{3}}{\cancel{14}_{2}} \times \frac{\cancel{7}^{1}}{\cancel{15}_{5}} = \frac{3\times1}{2\times5} = \frac{3}{10}$

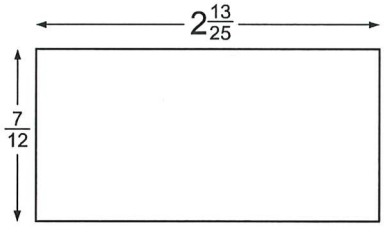
3 Solve and simplify $\frac{4}{9} \div 5 =$ $\frac{4}{9} \div \frac{5}{1} = \frac{4}{9} \times \frac{1}{5} = \frac{4}{45}$

4 Multiply $3\frac{4}{5} \times 2\frac{3}{8} =$ $\frac{3\times5+4}{5} \times \frac{2\times8+3}{8} = \frac{19}{5} \times \frac{19}{8} = \frac{361}{40} = 9\frac{1}{40}$

5 Solve and simplify $\frac{5}{6} + \frac{7}{15} =$ $\frac{5}{6} + \frac{7}{15} = \frac{25}{30} + \frac{14}{30} = \frac{39}{30} = \frac{13}{10} = 1\frac{3}{10}$

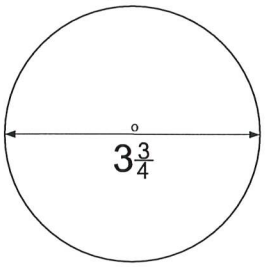
6 Solve and simplify $4\frac{2}{3} - 3\frac{2}{5} =$ $\frac{4\times3+2}{3} - \frac{3\times5+2}{5} = \frac{14}{3} - \frac{17}{5} = \frac{70}{15} - \frac{51}{15} = \frac{19}{15} = 1\frac{4}{15}$

7 What is the area of the rectangle below. All dimensions are given in cm.



$\frac{2\times25+13}{25} \times \frac{7}{12} = \frac{\cancel{63}^{21}}{25} \times \frac{7}{\cancel{12}_{4}} = \frac{147}{100} = 1\frac{47}{100}$

8 What is the area of the circle? All dimensions given in cm.



Radius $\frac{1}{2} \times \frac{3\times4+3}{4} = \frac{1}{2} \times \frac{15}{4}$

$= \frac{15}{8} = 1\frac{7}{8}$ cm

Area $= \pi r^2$

$= \frac{15}{8} \times \frac{15}{8} \times \pi = \frac{225\pi}{64} \approx 11.044\,66167$

$\approx 11\ cm^2$

PERCENTAGES AND DECIMALS

1 Write 7% as a decimal.

0·07

2 Write 0.78 as a percentage.

78%

3 Change $\frac{2}{5}$ into a percentage.

40% $\frac{2}{5} \times 100 = 40$

4 Change $\frac{3}{4}$ into a decimal.

$\frac{3}{4} = \frac{75}{100}$ (×25)

5 What is 24% of 260?

$\frac{24}{100} \times 260 = 62.4$

6 Bill, Charlie and Dave try to save money in their savings account.
Bill earns £2400 a month and has a ratio of spend: savings of 3:2.
Charlie earns £2200 each month and saves $\frac{5}{8}$ths of this amount.
Dave spends 65% of the £2600 that he earns each month.
Who saves the most money?

Bill 2400 ÷ (3+2) = 2400 ÷ 5 = £480
2 × £480 = £960

Charlie $\frac{5}{8} \times 2200 = £1375$

Dave $\frac{65}{100} \times 2600 = £1690$

7 A coat was in a sale where you saved 40%. The sale price of the coat was £120. How much was the original price of the coat?

100 − 40 = 60 $\frac{120}{60} = 2$ 2 × 100 = 200 £200

8 Gabriella puts £250 into her savings account and leaves it there for three years. Her bank account pays 4.8% interest per annum. How much money is in her bank account at the end of the three years?

$CI = PR^T$
$= 250 \times 1.048^3 = 287.755648$
$\approx £287.76$

9 Round 43.647 to three significant figures.

43·6

10 Round 382.9289 to two decimal places.

382·93

FACTORS, MULTIPLES AND PRIMES

15 different rectangles

1 A rectangle has a perimeter of 64cm. Each length of each side is an integer.

a. How many different sized rectangles can be made with these parameters?

$64 \div 2 = 32$

$1 \times 31 = 31$ $2 \times 30 = 60$ $3 \times 29 = 87$ $4 \times 28 = 112$ $5 \times 26 = 130$

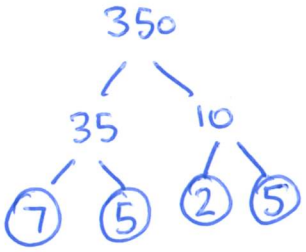
$11 \times 21 = 231$ $12 \times 20 = 240$ $13 \times 19 = 247$ $14 \times 18 = 252$ $15 \times 17 = 255$ $16 \times 16 = 256$

$7 \times 25 = 175$
$8 \times 24 = 192$
$9 \times 23 = 207$
$10 \times 22 = 220$

b. What is the range in the area of the rectangles?

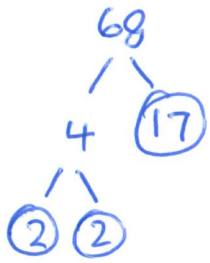
$(16 \times 16) - (1 \times 31) = 256 - 31 = 225$

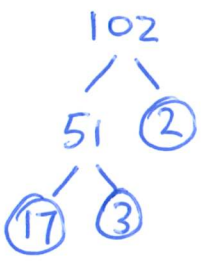
2 Write 350 as a product of its prime factors.

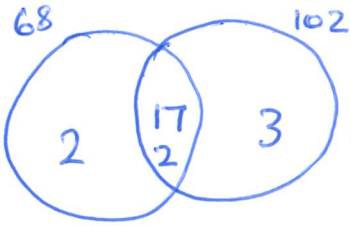


$350 = 2 \times 5^2 \times 7$

3 Bill and Ben both drive trains. When they can, they like to share their breaks. On Wednesday, Bills train takes a round trip that takes 68 minutes. Ben's train takes a round trip that takes 102 minutes. If they both start work and their first train sets off at precisely 5am, what time would they be both back at the station when they could meet up again?







Time is 8:24am

LCM $= 2 \times 2 \times 17 \times 3$
$= 204$
$= 3$hrs 24 mins

4 List all the factors of 48 that are less than 15.

1×48, 2×24, 3×16, 4×12, 6×8

1, 2, 3, 4, 6, 8, 12 are less than 15.

RECIPES

Charlotte loves baking. She wants to make some buns for a cake stall. She needs to make **45 buns** altogether.

Recipe for Chocolate Cupcakes with Buttercream (Serves 15)

225g unsalted butter, softened

225g caster sugar

4 medium free-range eggs

5 tbsp cocoa powder, dissolved in 3 tbsp boiling water

½ tsp vanilla extract

225g self-raising flour

150g unsalted butter, softened

300g icing sugar

1½ tbsp vanilla extract

1½ tbsp milk

Charlotte's dad is going shopping and Charlotte has asked him to get some ingredients. She has looked and sees that she only needs butter, caster sugar, self-raising flour and eggs.

How much of each ingredient does Charlotte's dad have to buy?

45 ÷ 15 = 3

225 x 3 = 675g butter

225 x 3 = 675 sugar

225 x 3 = 675 flour

4 x 3 = 12 eggs

Butter ..675.. g

Caster sugar ..675.. g

Self raising flour ..675.. g

..12.. eggs

TRIGONOMETRY AND PYTHAGORAS' THEOREM

1 ABC is a right-angled triangle. Find the length of the side AC.

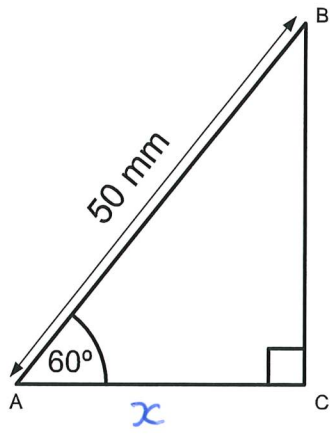
$S = \frac{O}{H}$ $C = \frac{A}{H}$ $T = \frac{O}{A}$

$\cos 60^\circ = \frac{x}{50}$

$\therefore \quad x = 50 \cos 60^\circ$

$= 50 \times \frac{1}{2}$

$= 25 \text{ mm}$



2 ABC is a right angled triangle. What is the length of side AC?

$AC^2 = 22.5^2 - 21.6^2$

$= 39.69$

$AC = \sqrt{39.69}$

$= 6.3 \text{ cm}$